JOURNAL OF MATERIALS SCIENCE 7 (1972) 615–620

Cavity Growth and Creep Rate taking into account the Change of Net Stress

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An expression is derived for the rate of void growth on grain boundaries taking into account the fact that the voids cause a reduction of the load-bearing area and thus an increase of the stress. Constant-stress creep tests were performed on OFHC copper at 425 °C and the data used to evaluate the proposed equation.

1. Introduction

Fracture of metals and alloys in creep or fatigue at elevated temperatures occurs generally by the linking of grain-boundary voids. The mechanism by which these voids nucleate and grow have been the subject of extensive studies.

Nucleation of the voids in creep is currently explained in terms of stress-concentrations at sliding grain boundaries [1]. Growth of voids may occur either by absorption of vacancies from the neighbouring grain boundaries [2, 3] or by grain-boundary sliding and associated dislocation mechanisms [4, 5].

Hull and Rimmer [3] assumed that vacancies migrate in a grain boundary towards a void owing to a chemical potential gradient caused by a tensile stress and obtained the expression for the growth rate:

$$\frac{\mathrm{d}r}{\mathrm{d}t} \simeq \frac{D_g\, z\, \Omega\, (\sigma - 2\gamma/r)}{2kTar}, \qquad (1)$$

where D_g is the grain-boundary atomic diffusion coefficient, z is the thickness of the grain boundary, Ω is the atomic volume, σ is the applied tensile stress, γ is the surface tension of the void, r is the radius of the spherical void, T is the absolute temperature, a is the void spacing and k is Boltzmann's constant. Void growth occurs by vacancy condensation when the applied stress is sufficiently large to overcome the tendency of the void to shrink under the action of surface tension, i.e.

$$\sigma > \frac{2\gamma}{r} \qquad (2)$$

Speight and Harris [6] assumed that the tensile stress causes generation of vacancies in the grain boundary at a constant rate. This gives the growth rate as

$$\frac{\mathrm{d}r}{\mathrm{d}t} \simeq \frac{K(1 - 4r^2/a^2)}{\ln(a/2r) - \frac{1}{2}(1 - 4r^2/a^2)} \qquad (3)$$

where

$$K = \frac{D_g\, z \cdot \Omega\sigma}{2kTr^2}, \qquad (4)$$

Equation 3 is similar to an expression which has been derived from an analysis of void nucleation by Harris [7]. Assuming that the voids are growing from a source of vacancies situated at the fixed distance $a/2$ away, Speight and Harris showed that

$$\frac{\mathrm{d}r}{\mathrm{d}t} \simeq \frac{K}{\ln(a/2r)} \qquad (5)$$

Hull and Rimmer's equation, 1, may be rewritten in terms of K:

$$\frac{\mathrm{d}r}{\mathrm{d}t} \simeq \frac{Kr}{a}. \qquad (6)$$

Speight and Harris [6] showed that equation 3 predicts higher growth rates than Hull and Rimmer's equation for very large and very small void spacings. Equation 5 gives larger growth rates than equation 6 for large void spacings and lower rates for advanced stages of growth. This will be further discussed later in this paper.

2. Stress Change during the Growth of Voids

The alternative equations for the growth rate of voids by vacancy condensation differ basically in the assumption of boundary conditions for diffusion. For the evaluation of these equations

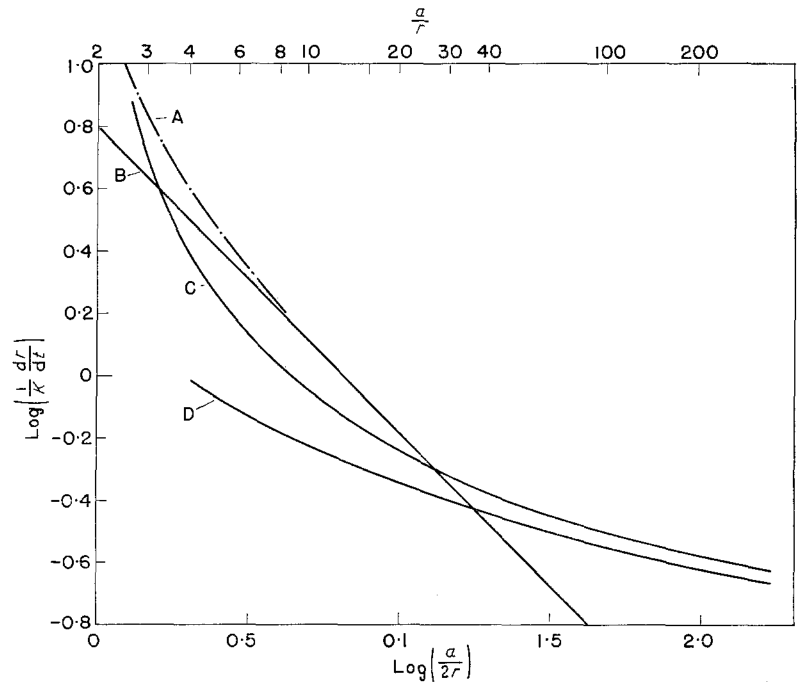

Figure 1 Growth rate of the voids as a function of their relative spacing. A. Present work; equation 8. B. Hull and Rimmer; equation 6. C. Speight and Harris; equation 3. D. Harris; equation 5.

it has been generally assumed that the stress remains unchanged during the test. In conventional constant-stress creep tests only the change of the external dimensions of the test specimen is compensated for. The reduction of the net section due to the voids is commonly neglected. If one ascribes a square section of a grain boundary area of size a^2 to each void then the net stress is given by

$$\sigma = \frac{\sigma_{:}\, a^2}{a^2 - r^2\pi} \quad (7)$$

where $\sigma_{:}$ is the nominal stress calculated from the load and the nominal cross section of the specimen, and r is the void radius. Substitution for the stress, equation 7, in equation 6 leads to the growth rate

$$\frac{\mathrm{d}r}{\mathrm{d}t} \simeq Ka \frac{r}{a^2 - r^2\pi} \quad (8)$$

To compare equations 4, 5, 6 and 8, $\log(1/K\,[\mathrm{d}r]/\mathrm{d}t)$ is plotted versus $\log(a/2r)$ according to Speight and Harris [6]. This is shown in fig. 1. Hull and Rimmer's equation gives a straight line. Equation 8, which considers the increase in net stress with increasing void size, results in an increase of the growth rate for large voids, i.e. small relative void spacing a/r. This trend is similar to that expressed by equation 3.

3. Relation between Void Volume, Strain, and Strain-Rate

The contribution of the voids to the total creep strain can be estimated by assuming that all atoms which diffuse from the voids in the opposite direction of the vacancies cause elongation of the specimen. A material which contains voids can be thought of being composed of small tetragonal prisms which contain a void in their centre. The prism has a base a^2 and a length l_0 as shown schematically in fig. 2. The rate of increase of the void volume, is thus related to the creep rate as follows:

$$\frac{1}{a^2 l_0}\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{1}{V_0}\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{1}{l_0}\frac{\mathrm{d}l}{\mathrm{d}t} = \dot{\epsilon}_V \quad (9)$$

Substitution of equation 8 into equation 9 gives the creep rate as a function of the strain caused by the void:

$$-\dot{\epsilon}_V = \frac{C_1}{C_2\,\epsilon_V^{-1/3} - \epsilon_V^{1/3}} \quad (10)$$

where

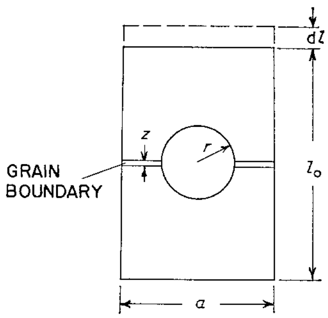

Figure 2 Definition of the elemental prism containing a void.

$$C_1 = \frac{2D_g\, z\cdot\Omega}{kT}\,\frac{\sigma_0}{a}\left(\frac{3}{4}\frac{a^2}{\pi}\right)^{-1/3} l_0^{-4/3} \quad (10a)$$

and

$$C_2 = \frac{a^2}{\pi}\left(\frac{3}{4}\frac{a^2}{\pi}\right)^{-2/3} l_0^{-2/3} \quad (10b)$$

The void spacing, a, can be either calculated from equation 10b:

$$a = C_2^{3/2}\cdot\pi^{3/2}\left(\frac{3}{4\pi}\right) l_0 \quad (10c)$$

or from combination of equations 10a and b:

$$a = \left(\frac{D_g\, z\, \Omega\, \pi\, \sigma_0}{kT}\,\frac{3}{2}\,\frac{C_2^{\,2}}{C_1}\right)^{1/3} \quad (10d)$$

According to equation 10 the creep rate $\dot{\epsilon}_V$ approaches infinity when

$$C_2\, \epsilon_V^{-1/3} - \epsilon_V^{1/3} = 0$$

giving a maximum strain due to the voids

$$\bar{\epsilon}_V = K_2^{2/3} \quad (10e)$$

This relation can be used for an estimate of the magnitude of C_2.

4. Experimental Procedure

To check some of the relations which were derived in the previous section, constant-stress creep tests on OFHC copper were performed at 425°C in an argon atmosphere. The specimens had a 2 in. (5 cm) gauge length and a diameter of $\frac{1}{4}$ in. (0.6 cm). Annealing of the test specimen was accomplished in the creep-testing machine by holding at the test temperature for 1 h before load application. The stress was maintained constant by means of an Andrade-Chalmers type cam. The creep elongation was recorded throughout each test. Creep curves which were used for evaluation are shown in fig. 3.

For the definition of strains and strain rates it is assumed that accelerated creep is primarily

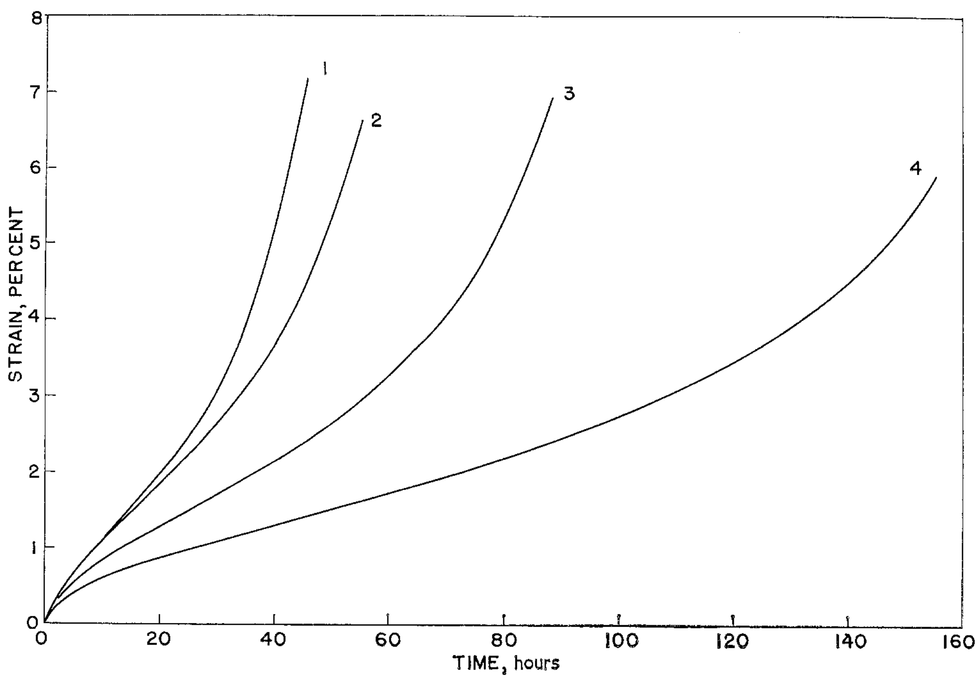

Figure 3 Creep curves for OFHC copper tested at 425°C. Test numbers refer to table I.

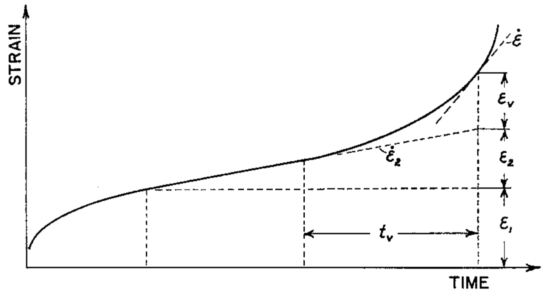

Figure 4 Definition of strain and strain rate for evaluation of equation 10.

caused by void growth which superimposes on second stage creep. This is schematically illustrated in fig. 4. The strain contribution by the voids is thus given by

$$\epsilon_V = \epsilon_{total} - (\epsilon_1 + \epsilon_2) \quad (11a)$$

Similarly:

$$\dot{\epsilon}_V = \dot{\epsilon} - \dot{\epsilon}_2 \quad (11b)$$

The constants C_1 and C_2 of equation 10 were calculated from selected points of the creep curves. The data are summarised in table I.

TABLE I Test data

Test No.	Stress (psi) or (kgf/cm²)	Time to fracture (h)	$C_1 \times 10^4$ (h^{-1} cm^{-1})	$C_2 \times 10^2$
1	5310 (372)	45	3.03	9.60
2	4700 (330)	55	2.43	9.55
3	4290 (300)	88	2.09	9.77
4	3880 (272)	15.6	2.32	8.52

5. Discussion

Fig. 5 shows curves for creep rate versus creep strain which were replotted from the creep recordings. Data points which were calculated from equation 10 using the constants given in table I are also plotted for comparison. In general, the calculated data compare favourably with the experimental ones over a wide range of strain. At very small and very large strains the calculated creep rates are higher than the experimentally observed ones. This may be due to the simplifying assumption implied in this evaluation, i.e. all voids are of the same size and grow at the same rate; also linking of voids in advanced stages of cavitation has been neglected.

The validity of equation 10 can be further checked by calculating the theoretical void spacing and comparing it with the observed one. To use equation 10c it is assumed that l_0 corresponds to the measured mean grain size of 7.9×10^{-3} cm. Selecting $C_2 = 0.0955$ from test no. 2 gives a void spacing, a, of 3.1×10^{-4} cm. This compares favourably with the metallographically observed void spacing of 3.5×10^{-4} cm.

The void spacing may also be calculated from equation 10d without a knowledge of l_0. However, it is necessary to estimate the values for the

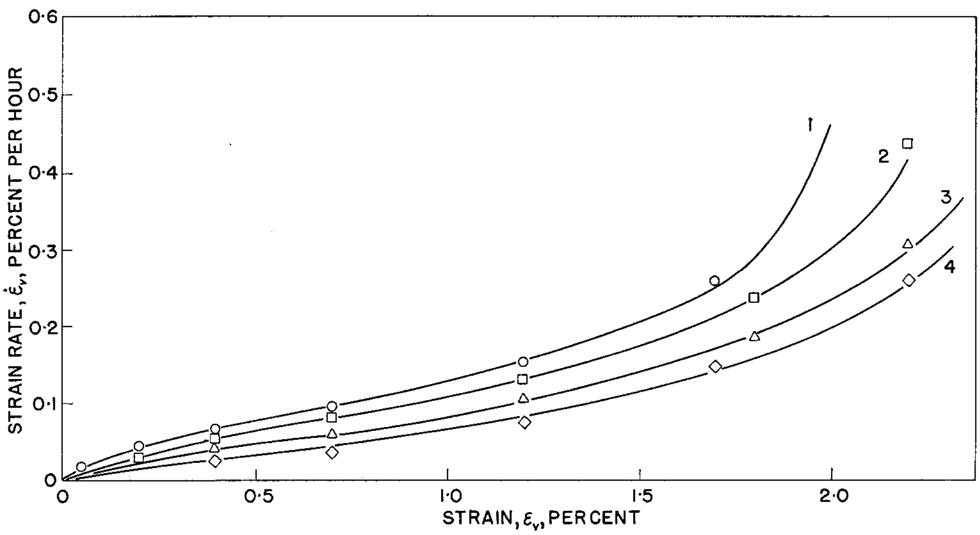

Figure 5 Strain rate versus strain for third stage creep.

grain-boundary thickness, the diffusivity, and the effective atomic volume. For simplicity the values suggested by Cottrell [8] are used here, i.e. $D_g z = 3 \times 10^{-15}\,\text{cm}^2\,\text{sec}$, $\Omega = 1.2 \times 10^{-23}\,\text{cm}^3$. The data for test no. 2 are: $\sigma = 3.24 \times 10^8$ dyn/cm^2, $kT = 8.27 \times 10^{-14}$ erg, $K_2 = 0.0955$, $C_1 = 2.43 \times 10^{-4}/3600 = 6.76 \times 10^{-8}$ sec^{-1} cm^{-1}. Thus, the void spacing calculated from equation 10d is 4.5×10^{-4} cm which comes close to the previous values for a.

The relation between void volume and time of void growth, t_V, is obtained by integration of equation 8, hence

$$V^{2/3} - C_3 V^{4/3} = C_4\,\sigma_0\,t_V \tag{12}$$

where

$$C_3 = \frac{\pi}{2a^2}\left(\frac{3}{4\pi}\right)^{2/3} \tag{12a}$$

and

$$C_4 = \frac{2}{a}\left(\frac{3}{4\pi}\right)^{-2/3}\frac{D_g\,z\,\Omega}{2kT} \tag{12b}$$

For large void spacings, i.e. very small voids, the second term in equation 12 becomes negligibly small and thus

$$V \simeq (C_4\sigma_0 t_V)^{3/2} \tag{12c}$$

This is the same form of equation as that derived from the Hull and Rimmer equation. Initially the void volume should change according to $V \propto t^{3/2}$. As void growth progresses, the second term of equation 12 becomes significant and must thus be considered.

The change of specific gravity with creep time has been measured by several authors. Gittins [9] found that the total cavity volume of copper changed according to $V \propto t^{3/2}$. Bowring, Davies, and Wilshire [10] used a definition of strain similar to that given in equation 11a. The authors evaluated data for a magnesium alloy and a nickel alloy and found that for both the time dependence of the total void volume was expressed by $V \propto t^{1.7}$.

The good agreement between the experimental data and equation 12c is considered by several authors as fortuitous because of the assumption that the number of voids remains constant throughout the test. Oliver and Girifalco [11] determined the number of voids as a function of time and temperature. The authors found that an incubation period occurs, after which the number of voids increases rapidly and then approaches asymptotically a constant value. Oliver *et al* reached the conclusion that void nuclei are pre-existent features of the grain boundaries.

Gittins found that his void count on copper creep specimens could be represented by the relation $N \propto t^{1/2}$. Ratcliffe and Greenwood [12] explain their data on density changes of magnesium by assuming that the number of voids is linearly increasing with time.

Both, Gittins' parabolic relation and Greenwood's linear relation could be interpreted as being valid for segments of the sigmoid curve observed by Oliver and Girifalco for the variation of void density with time.

6. Summary and Conclusions

1. The Hull and Rimmer equation for the growth of voids by vacancy condensation has been modified to take into account the reduction of the net area of cross section by the voids. This leads to a steadily increasing stress in nominally constant stress tests.

2. The growth rate of voids has been related to the creep rate, assuming volume constancy of the material. This makes it possible to correlate the equation for void growth with creep data in third stage creep.

3. Creep tests were performed on copper. The data were evaluated in terms of the modified equation. The results indicate that accelerated creep is largely due to the growth of voids and the resulting increase in net stress.

4. Calculated values for the void spacing agree favourably with measured ones.

Acknowledgements

The author thanks Mr G. Sidla for performing the creep tests. The financial support received from the Graduate Faculty of the University of Alberta is gratefully acknowledged.

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Received 8 October and accepted 16 November 1971.